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Patents ADP number *(if you know it)*
870407400l

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4. Title of the invention
Digital Modulation Waveforms for use in Ranging Systems

5. Name of your agent *(if you have one)*
Harrison Goddard Foote

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Digital Modulation Waveforms for use in Ranging Systems

Field of the Invention

The invention relates generally to the field of navigation and positioning systems, including satellite systems, and specifically to improvements in the waveforms and spectra that are emitted by such systems and satellites and to techniques for receiving such improved waveforms.

Satellite Positioning Systems (SPS) rely on the passive measurement of ranging signals broadcast by each satellite, or ground-based or airborne equivalent, in a specific constellation or group of constellations. Typically, the time reference for each of the broadcast ranging signals are synchronised by a clock flown on board the satellite or equivalent, although there are some exceptions to this technique. The on-board clock is used to generate a regular and usually continual series of events, often known as 'epochs', whose time of occurrence is coded into a random or pseudo-random code (known as a spreading code). As a consequence of the pseudo-random or random features of the time epoch encoding sequence, the spectrum of the output signal is spread over a frequency range determined by a number of factors including the rate of change of the spreading code elements and the waveform used for the spreading signal. In almost all cases of interest, the spreading waveform has been rectangular leading to a spectrum envelope whose shape follows a sin(x)/x function. This spectrum shape is correct whether the spectrum is a power spectral density or the modulus of a complex spectrum.

The ranging signals are modulated onto a carrier signal for transmission to the passive receivers adapted to receive these signals. Applications are known which cover land, airborne, marine and space use. Many modulation techniques are possible, but as is known in the art, it is binary phase shift keying which has been employed by which the carrier signal is of constant magnitude but alternates between phase states that are 180° apart. At least two such signals may be modulated onto the same carrier in this way but with phase states in quadrature. The resulting carrier signal retains its constant envelope but has 4 phase states depending upon to two independent input signals. The two modulating signals do not need to have the same carrier magnitude. The constant carrier magnitude of the combined signal is maintained, although the phase states are not 90° except in the equal signal magnitude case. Figure 1 illustrates thus case for unequal quadrature carrier magnitudes.

Figure 1 shows an Argand diagram for the phases of a signal at a carrier frequency (not specified). The signal is composed of two components in phase quadrature orientated along the I and Q directions. The signal component along the I direction is approximately twice the magnitude of the Q component. The modulation of each component is binary phase shift keyed (BPSK). The table included in figure 1 contains the relationship between the phase state (1 to 4 inclusive) and the binary states (+ or -) of each of the signal components.

As an example in the known art, the CA code signal emitted by the GPS Satellite Navigation System is broadcast on a frequency of 1575.42MHz, known as L1, with a spreading code rate (chipping rate) of 1.023MHz and has a rectangular spreading waveform. It is categorized as BPSK-R1. For GPS, the signal broadcast by the satellites on the L1 frequency has a second component in phase quadrature, which is known as the precision code (P(Y) code) and made available to authorized users only. The P(Y) signal is BPSK modulated with a spreading code at 10.23MHz with a magnitude which is 3dB lower in signal power than the CA code transmission. Consequently, the Q signals have a magnitude which is 0.7071 (-3dB) of the magnitude of the I signal components. The phase angles of each state are, therefore, ±35.265° in relation to the ±I axis (phase of the CA code signal as specified in ICD GPS 200C).

The satellite constellations typically contain 10's of satellites often in similar or similarly shaped orbits. The transmissions from each satellite are on the same nominal carrier frequency in the case of code division access satellites (such as GPS) or on nearby related frequencies such as GLONASS. The satellites transmit different signals to enable each one to be separately selected even though several satellites are simultaneously visible. The visibility charts for GPS show that there are times of the day and locations where in excess of 10 satellites may be visible at the same time. The joint constellations of GPS and Galileo will provide simultaneous satellite visibility in excess of 22 satellite signals.

The signals from each satellite, in a CDMA system like GPS, are selected from one another by means of the different spreading codes and/or differences in the spreading code rates, which each satellite broadcasts; whereas in Glonass the satellite signals are separated from one another by virtue of differences in transmission frequencies. Nevertheless, there still remains residual interference between all the different signals.

There are several different classes of interference. We identify at least two categories – which are related to the signal spectrum differences and those associated with code difference usually established through the code family cross and auto correlation properties. Where the code families are different between 2 satellite systems, the cross correlation properties betweens the families is a descriptor of the mutual interference. The cross and autocorrelation functions are well known in the art. For the GPS satellite system, the CA code is formed from a family of 1023 length codes generated from a selected pair of 10-bit feedback shift registers in accordance with the method established by R Gold. This method not only specifies selected pairs of shift register sequences, but also prescribes a 3-level cross correlation function between family members.

When the codes come from different families and the transmission frequencies and/or spectra are different alternative means for establishing measures of mutual interference are required. Such tools have been developed in the form of a Spectral Separation Coefficients (SSC) linking two interfering signals. SSC's have their origins in the cross power spectral density $\Phi_{12}(\omega)$ defined as the product of two power spectral densities (PSD), $\Phi_2(\omega)$ and $\Phi_1(\omega)$ (see Pratt & Owen). The SSC between spectra 1 and 2 is defined as κ_{12}:

$$\kappa_{12} = \int_{-\infty}^{\infty} \Phi_1(\omega).\Phi_2(\omega).df \qquad 1.1$$

The values of $\Phi_2(\omega)$ and $\Phi_1(\omega)$ are normalized so that each represents a transmission with the energy of 1 watt:

$$\int_{-\infty}^{\infty} \Phi_1(\omega).df = 1$$
$$\int_{-\infty}^{\infty} \Phi_2(\omega).df = 1 \qquad 1.2$$

By means of Parseval's Theorem, the integrals of the PSD may also be written in terms of their auto-correlation functions (ACF):

$$\phi_1(0) = \phi_2(0) = 1 \qquad 1.3$$

Through this definition, there are also definitions for the self SSC for each spectrum:

$$\kappa_{11} = \int_{-\infty}^{\infty} \Phi_1(\omega).\Phi_1(\omega).df$$
$$\kappa_{22} = \int_{-\infty}^{\infty} \Phi_2(\omega).\Phi_2(\omega).df \qquad 1.4$$

The ratio between the SSC between two spectra and the self SSC for one of the signals gives a direct measurement of the energy which leaks from one signal through a receiver with a filter matched to the spreading waveform of the other member of the pair.

The purpose of using the spectral separation coefficients is to quantify the effects of using one signal in comparison with another for the purpose of establishing the level of mutual interaction. The spectral separation coefficient may, therefore, be used as a metric in selecting different spreading code waveforms to control or minimize the levels of mutual interference between two different satellite

signals. This may be an important system design consideration in navigation and communication satellite systems.

Brief Description of the Invention

The invention provides improvements to a satellite-type signal which is modulated by a sub carrier as in known in the art as BOC (binary offset carrier) or LOC (linear offset carrier) but using a stepped multi-level waveform. In one preferred embodiment, the stepped waveform is an approximation to a sinusoidal sub carrier modulation. One method of realization is through a multi-angle phase modulation in some cases.

The term "satellite-type" signal is intended to encompass not only satellite signals themselves, e.g. as used in satellite based ranging systems, but also any other similar or related signals used in location and/or navigation equipment in other environments, for example on the ground or in an airborne vehicle.

The invention also provides for a satellite-type signal which is modulated by a stepped or binary sub carrier which is itself the product of a plurality of binary or stepped sub carrier modulation signals.

The invention also provides for a satellite-type signal which is modulated by a stepped or binary sub carrier sequence containing a random or pseudo-random code as a sub sequence. The code signal is constrained to repeat within a spreading pulse or a plurality thereof, so replacing the binary offset carrier with a spread spectrum waveform.

Thus, in more general terms, whereas it is known in the prior art to use a digital sub-carrier having only two amplitude levels with constant temporal intervals between amplitude transitions, a sub-carrier in a satellite-type signal according to the invention has a digital waveform in which the amplitude has more than two values and/or in which the time between successive transitions has more than one value.

The invention may facilitate the generation of signals which exhibit low coefficients of spectral separation with other signals at near frequencies due to the low levels of the out-of-band sidelobes.

At least for some of the signal versions according to the invention, it is possible to obtain signals which have a constant magnitude.

A further aspect of the invention provides for a spreading waveform which is a stepped or profiled wavefom (multi-level) in contrast to the well known prior art in binary waveforms.

The invention also provides for systems in which signals or waveforms according to the invention are generated transmitted or received.

The invention also provides for receiver apparatus incorporating demodulation means for the signals listed above in this invention.

The invention also provides for navigation signal generators used to generate the signals of this invention either for use in satellite applications or in ground or airborne transmitters.

The invention also provides for signal generators which are used to make replicas of the signals of this invention. Such replica generators are often found in receivers and signal simulators.

The invention also provides for the use of magnetic or other computer storage media incorporating instructions for generating or demodulating signals incorporating the signal inventions listed above.

Detailed Description of the Invention

It is a primary purpose of this invention to provide improved waveforms for the modulation of satellite signals for use in satellite based ranging systems. Such systems are the heart of satellite location and navigation systems. As is well known to those skilled in the art, all such systems may also be considered to have significant ground components (transmitters) not only for the support of the satellite constellation but also as 'local' components of the system. The scope of this invention is therefore not limited to satellite based ranging systems but also those operative with entirely ground based transmitters or a combination of ground, airborne and satellite transmitters. Although the invention is directed to satellite location and/or navigation systems, to those skilled in the art, it will be clear that the concepts described herein are equally applicable to communication systems which encompass a ranging component. It is intended that such systems are also considered to be within the scope of this invention.

The invention is specifically concerned with a part of the payload of a satellite constellation used for generating the navigation signals. As in known in the art, such signals are often generated in a binary form and these are used, after multiplication, to modulate the carrier components transmitted. The navigation signals include the spreading waveform, the coded ranging signal, the data message including encryption and error correcting parts to the message as required. Examples of such signals can be found in the known art such as in the GPS and Glonass systems. GPS CA code is a ranging signal with a chipping rate of 1.023Mcps and a sequence length of 1023 binary elements, repeating every 1ms. The data rate imposed on this signal is at a rate of 50bps with a message broken into 5 sub-frames of data of 300 bits each. Four of the sub-frames repeat every 30 seconds whilst the fifth sequences through 25 different versions, each one containing the almanac of one of the satellites in the constellation. The ranging sequence is convolved with a rectangular pulse which lasts for a duration exactly equal to the time between changes in the ranging code signal. The rectangular pulse is known as the spreading waveform. An illustration of such a waveform is shown in figure 2. In figure 2, the spreading waveform, A, is a rectangular in shape. The ranging code is a series of signal states (10101001100010 in the shown example) whereas the data signal is shown in 2 states only (10 in the shown example) with the transition in the data signal from 1 to 0 taking place exactly at the 10 transition within the ranging code during the 11000 sequence of states. This causes the ranging sequence to be inverted during the '0' state of the data signal. Consequently, the transmitted sequence is 10101001111101. The composite resulting waveform shown in the top line of figure 2 is then modulated onto the satellite carrier using binary phase shift keying (BPSK) wherein a '0' state is transformed into a '+1' signal and modulated as a 0° phase and a '1' state is transformed into a '-1' signal and modulated as a 180° phase angle.

The spectrum resulting from BPSK modulation of a carrier signal has spectral components that extend well beyond the chipping frequency. In some cases the range of frequencies emitted from the satellite may be limited by various filters, such as the output multiplex filter connecting the HPA (High Power Amplifier) to the antenna system or by other filters. Furthermore, the navigation signal receiver also contains frequency filtering components such as antenna and receiver filters, often used to limit the receiver sensitivity to unwanted signals.

In the description of this invention, the spreading waveform has been specifically separated from the coded ranging signal. This is purposeful as the improvements described in the invention relate specifically to the design of the spreading waveform and not to the ranging code. The invention identifies the spreading waveform, which has hitherto been only binary in satellite and ground based ranging systems, as a mechanism for controlling the spectrum of the ranging signal emissions including changing the frequency at which the peak energy is transmitted and through such spectrum control, bringing improvement the intersystem interference metrics, for example through the spectral separation coefficients. We now describe five specific embodiments of the invention with different spreading waveforms as illustrations of the concept.

In the first embodiment of the invention, the spreading waveform is a multi-level signal. This is illustrated in figure 3 in which a 5-level spreading signal is shown replacing the BPSK signal of the known art. The BPSK signal takes carrier phase values of 0° and 180° corresponding to signal values of the composite spreading waveform, ranging code and data signal of (+1, -1). For the 5-level waveform, the levels may be equi-spaced but do not need to be. In the example of figure 3 the signal levels are taken from the set ($+1$, $+1/\sqrt{2}$, 0 $-1/\sqrt{2}$, -1). The levels are the resolved amplitudes along

one of the axes, in a careteslan co-ordinate system, of a rotating vector at angles every 45° around a circle with a (0,0) origin and having unit magnitude. On its own, this signal would not be very useful in a satellite transmission system because it does not have constant magnitude. However, in combination with a similar signal in phase quadrature, a constant magnitude signal can be formed. There are some constraints to be satisfied if a constant modulus signal is required. The rules are:

'+1' or '–1' on one phase (I) can only occur when a '0' occurs on the Q phase, and *vice versa*

'±1/√2' must occur on both phases (I and Q) simultaneously.

Another way of viewing the invention is to consider the 8-phase signal (8-PSK) represented by the rotating vector with angles every 45° around the unit circle in an Argand Diagram. Such a signal is capable of transmitting for each symbol 3 independently determined bits of information (in accordance with Shannon's Information Theory). The present formulation is a degenerate usage of the 8 level symbol in which the capability to carry one bit of information has been exchanged, through the rules above, for some control over the transmitted waveform and thus its spectrum. The waveforms for the I and Q components are thus built from the following signal element sequences:

I phase - (+1/√2, +1, +1/√2, 0) representing a +1 signal
I phase - (-1/√2, -1, -1/√2, 0) representing a -1 signal
Q phase - (+1/√2, 0, -1/√2, -1) representing a +1 signal
Q phase - (-1/√2, 0, +1/√2, +1) representing a -1 signal.

Any combination of I or Q signal sequences can be chosen from the above set within the constraint of a constant magnitude carrier signal, computed as (I^2+Q^2). It will be clear to those skilled in the art, that there are many other equivalent sets of sequences which may be chosen from the set of 5-levels satisfying the criteria of constant carrier envelope. Such sequences are included within the scope of the invention.

Several additional further variations in the assignment of code and data states to the phase locations are possible. One example of this may be generated through rotation of the phase states on the Argand diagram - representing the assignment of carrier phases. A re-assignment from the angles set (0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°) to the angles set (22.5°, 67.5°, 112.5°, 157.5°, 202.5°, 247.5°, 292.5°, 337.5°) is a simple rotation through 22.5° and, therefore, causes no change in power spectral density or spectrum modulus. However, the number of required amplitude levels in the I and Q signals changes from 5 to 4. The resulting waveforms for the I and Q components are built, in this case, from the following signal element sequences:

I phase - (+cos(67.5°), +cos(22.5°), +cos(22.5°), +cos(67.5°)) representing a +1 signal
I phase - (-cos(67.5°), -cos(22.5°), -cos(22.5°), -cos(67.5°)) representing a -1 signal
Q phase - (+sin(67.5°), +sin(22.5°), -sin(22.5°), -sin(67.5°)) representing a +1 signal
Q phase - (-sin(67.5°), -sin(22.5°), +sin(22.5°), +sin(67.5°)) representing a -1 signal.

It may be noted that the I and Q signal element sequences for both the cases described above are orthogonal over the duration of one spreading pulse (chip) as shown in A of figure 2. Clearly, other rotations are possible and all will yield orthogonal signal element sets.

A further change may be envisaged in which the target carrier envelope is not constant. The requirement of constant envelope is beneficial to the performance of some forms of satellite based high power amplifiers (HPA's). In other cases, this may be a critical requirement and could be relaxed. Other possible envelopes of carrier level could be elliptical or polygon on shape. The elliptical shape is of interest because it arises from unequal magnitudes for I and Q signal elements.

Those skilled in the art will recognize that there are other variations within the scope of the invention which produce alternative usable arrangements. These include the use of unequal angular spacing (not 45° for 8 PSK systems) and the application of uneven spacing of the time samples.

Another means of understanding the invention as embodied above is through a state chart. This shows the sequence of states depending upon the I and Q states for which transmission is required

and the sequence of states required to control the spectrum of the transmission. Figure 4 shows an Argand diagram with 8 phase states identified at regular 45° increments starting at 22.5°, each phase state is numbered 1 to 8 inclusive. Table 1 shows the sequence of phase states required for each I and Q state signal element.

I	Q	t_1	t_2	t_3	t_4
+1	+1	2	1	8	7
-1	+1	3	4	5	6
+1	-1	7	8	1	2
-1	-1	6	5	4	3

Table 1 – Sequence of States for 8-PSK I & Q signal elements

In a second aspect of the invention, it is evident that the signal element sequence are sections (specifically half cycle sections in the aspect of the invention disclosed above) from a sampled sinusoid. The concept can, therefore, be extended to include a multiplicity of such samples. Those variants, which appear to be useful, include the cases with samples from a finite number of half cycles. An example is given in table 2 based on the phase state diagram of figure 4 for samples from 1½ cycles (or an arbitrary number of half cycles) of the sinusoid waveform. It is worthwhile to identify the sinusoid, from which these samples are taken, as the 'basis waveform' since there are other choices for the basis waveform which lead to useful application classes.

I	Q	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
+1	+1	2	1	8	7	6	5	4	3	2	1	8	7
-1	+1	3	4	5	8	7	8	1	2	3	4	5	6
+1	-1	7	8	1	2	3	4	5	6	7	8	1	2
-1	-1	6	5	4	3	2	1	8	7	6	5	4	3

Table 2 – Sequence of States for 8-PSK I & Q Signal Elements with 1½ cycles of sub-carrier modulation

It will seen that there are 4 time samples for each ½ cycle of the waveform which replaces the spreading waveform of figure 2 denoted by 'A'. The stepped sinusoidal sampled waveform may be viewed as a sub-carrier modulation of the basic spreading waveform. The number of time samples and independent information bearing channels is related to the number of phase states which the carrier signal has in its representation. Although the examples above have used 'powers of 2' this is not necessary. A 6-PSK carrier signal can be used to carry 2 independent information bearing binary channels. In this case only 3 signal element samples are required per transmitted code chip (spreading waveform element 'A').

In order to show the benefits of this aspect of the invention, the spectrum of a BOC(2,2) and its stepped sinusoidal counterpart, denoted BOC8(2,2), are illustrated in figure 5. The vertical scale is logarithmic (calibrated in decibels) whilst the horizontal scale is in frequency offset (MHz) from the carrier frequency. The (upper) dotted curve is the binary offset carrier modulated code signal spectral envelope whilst the lower (solid) curve is a BOC8(2,2) spectrum using a stepped sinusoid with 5 levels, as described hereinbefore. This is realized using an 8 phase carrier modulation. It is evident that the BOC8(2,2) signal exhibits lower sidelobe levels than the binary sub-carrier equivalent in BOC(2,2). This is of benefit when it is necessary to use the spectrum in the region of the sidelobes for the transmission of another satellite signal (or another satellite service). The lower sidelobe levels add to the isolation between such signals. This isolation is measured using spectral separation coefficients (SSC's).

Replacement of the stepped sinusoid with a rectangular wave with duration of each element equal to a ½ cycle of the sinusoid is a known form in the art. It is known as 'Binary Offset Carrier' or BOC as an acronym. There are usually 2 further attributes associated with the BOC description which relate to the frequency of the code chipping rate and to the frequency of the offset sub-carrier. BOC(2,2) consequently is interpreted as a waveform with a 2.046MHz chipping rate and a 2.046MHz offset sub-carrier. This arrangement has exactly two ½ cycles of the sub-carrier signal for each code element (chip). A further arrangement is known in which the sub-carrier is a pure sinusoid (rather than a

stepped equivalent). This class is known as LOC(m,n) modulation – for Linear Offset Carrier. Whilst this offers good spectral properties, there are disadvantages in aspects of power amplification as the envelope of the carrier signal is not constant.

In a further aspect of the invention, the sub-carrier signal may be formed from a set of signal elements comprising a multi-level representation of a basis waveform which is further modulated by a second basis waveform. In general, a multiplicity of layers of modulated basis waveforms can be used to construct a suitable sub-carrier waveform, though in practice, it is rarely of benefit to use more than 2 basis signal thus nested. As examples of this aspect of the invention, figure 6 contains the waveform for the spreading waveform of each code in which there is a second sub-carrier modulation. In figure 6, as an illustration of the invention the sub-carrier basis waveform are binary. The extent of the time duration in figure 6 is 512 samples and exactly matches the duration of one code element duration (chip). There are two basis waveforms – the first (a) contains 4 ½ cycles of a sub-carrier shown as the dotted waveform. If this were the only sub-carrier component, the modulation would be a BOC(2x,x) type, where x is the frequency of the code rate (chipping rate). However, there is a second sub-carrier (b) as shown in the solid curve (also binary in this case) that modulates the first sub-carrier signal. There are 16 ½ cycles of the second sub-carrier in the chip duration (512 samples). As a result of the modulation (multiplication) of the two sub-carriers, the resulting waveform has phase reversals in the (b) sub-carrier whenever there is a sign reversal in the (a) sub-carrier. This is clearly evident in the figure(6). The resulting modulation is denoted Double BOC, or DBOC. In the case of figure 6 the modulation is DBOC(8x,(2x,x)). The main energy is concentrated around frequencies ±8x from the carrier signal, with a BOC like double humped spectrum.

A typical spectrum is shown in figure 7 for a $DBOC_8(16,(2,2))$ type signal. A $BOC_8(2,2)$ spectrum is also shown in Figure 7 for comparison purposes. The spectra shown have been made using a previous aspect of the invention in combination with the Double BOC concept – that of using stepped approximations to sinusoidal sub-carrier modulation. The waveforms for the I & Q modulations for the spectrum of figure 7 are given in figure 8. These illustrate the use of the stepped sinusoidal modulation on the first sub-carrier (a) whereas the second sub-carrier (b) is used in a binary form. In the double modulation arrangement, the magnitude of the second sub-carrier (1) is multiplied by the magnitude of the first sub-carrier (a – stepped sinusoid).

A further feature of the invention is illustrated in figure 8 in that a cooperative set of signals to the $DBOC_8$ signal but only using BOC_8 modulation (stepped sinusoid) is used on the Q channel and yet still allows a constant envelope carrier signal to be emitted from the satellite.

A final feature of the invention is the use of a random sequence to act as a sub-carrier instead of the binary or stepped sinusoidal modulations. The use of additional sequences to that of the main spreading code has hitherto been limited to use as a tiered code which changes state after every complete code repetition interval. The GPS L5 codes are constructed in this manner using Neumann Hoffman sequences of length 10 or 20 to extend a 1ms code (of 10230 chips or elements) to 10ms or 20ms. The use of a sub code chip interval has not previously been considered. A complete sequence (a sub-sequence) is of duration one, or at most a plurality, of code chips. It fulfills a similar role to the sub-carrier modulation as described hereinbefore in that is controls the spectrum of the emissions. One feature of such a sub-sequence is that such sequences may be chosen to be common amongst a satellite constellation or a sub-set of the constellation. One such subset might be a group of ground transmitters providing a local element or augmentation to the space segment of the system.

Another purpose for the use of the sub-sequence is to improve the isolation between two satellite signals through spreading the energy of one signal over a wide spectrum.

The illustrations have used above have exclusively employed stepped and sampled versions of sinusoidal or binary waveforms for the sub-carrier modulating signals. In yet a further aspect of the invention, the amplitude steps and sample durations are varied for the purpose of steering a spectral null into a required frequency location. An additional use of the sample duration modulation and amplitude levels control is to meet another signal metric such as minimize the energy in the harmonics of the resulting sub-carrier modulation waveform. An illustration of the first of these aspects is observed in figure 5 in which addition spectral nulls appear in the $BOC_8(2,2)$ spectrum at approximately 6MHz and 10MHz offset from the carrier frequency. There are no such nulls in the

spectrum of the BOC(2,2) – using binary sub-carrier modulation. The appearance of the nulls is not accidental and the location of the nulls can be controlled by the relative magnitude of the steps for this case.

FIGURES

Phase State	1	2	3	4
I signal	-	+	+	-
Q Signal	+	+	-	-

State Table for modulation in figure 1

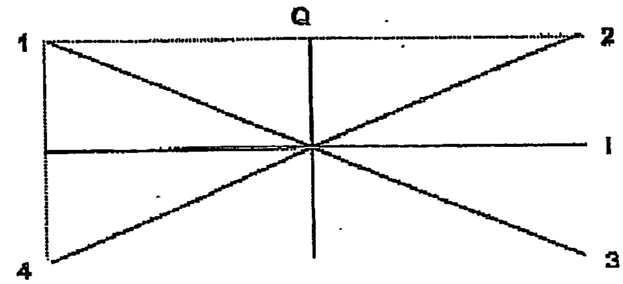

Figure 1 – Argand Diagram for two BPSK signals in phase quadrature – unequal signal levels

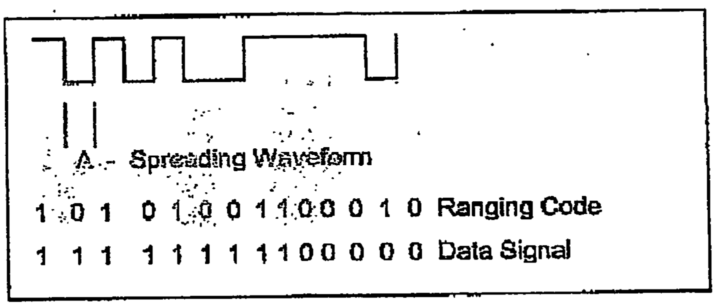

Figure 2 – Illustration of features of a combined ranging signal and data waveform

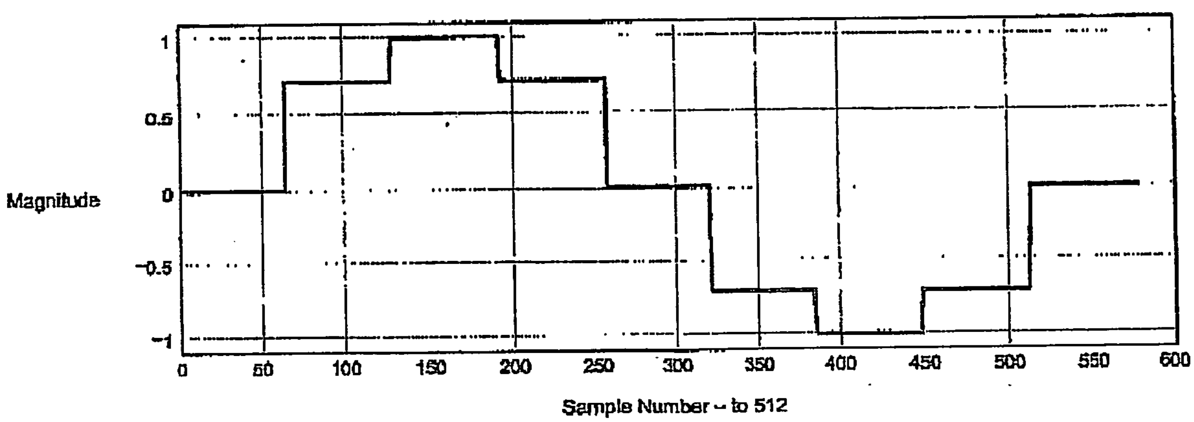

Figure 3 – Illustration of a 5-level stepped waveform
Amplitudes follow a sinusoidal distribution

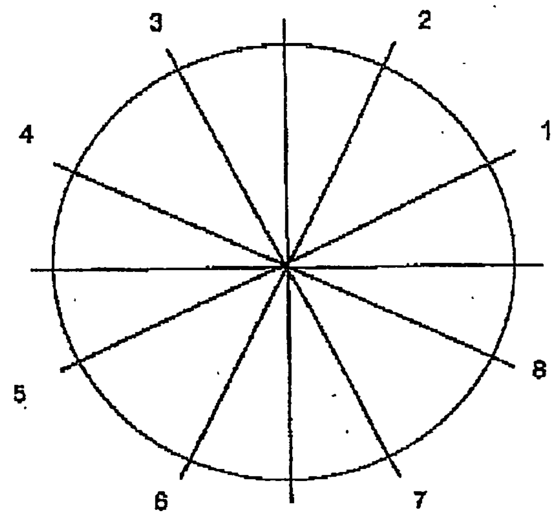

Figure 4 – Carrier Phase State locations for Table 1

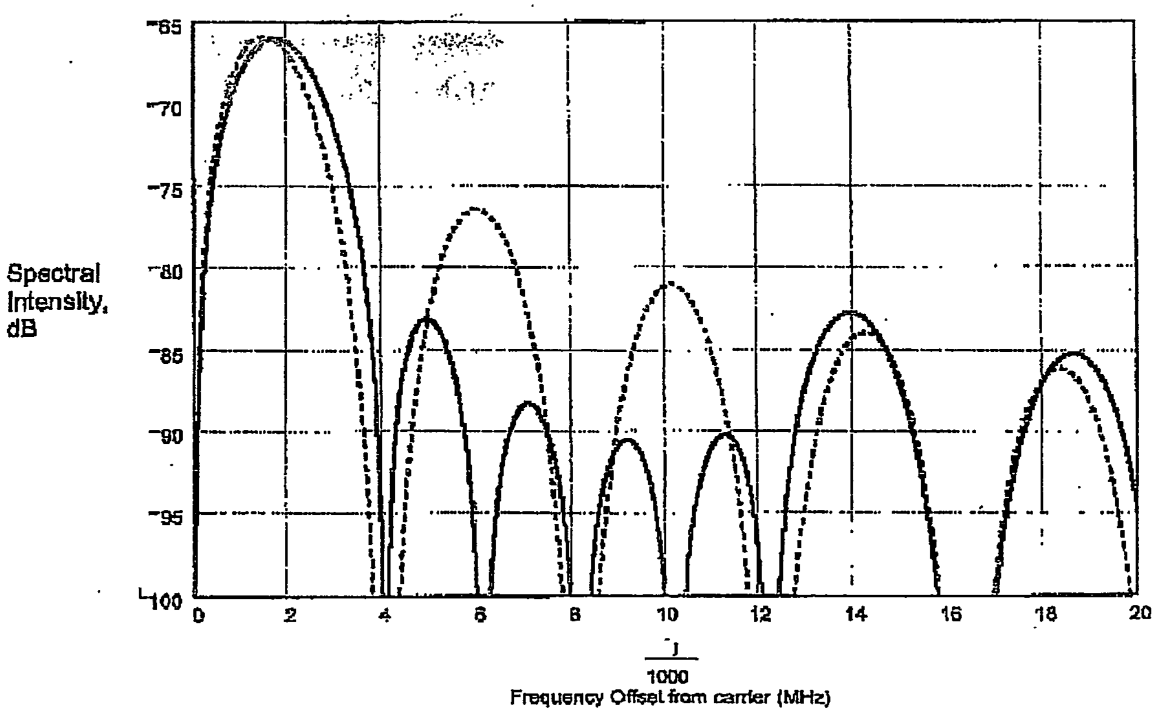

Figure 5 – Comparison of BOC(2,2) (dotted) and $BOC_8(2,2)$ (solid) Spectra

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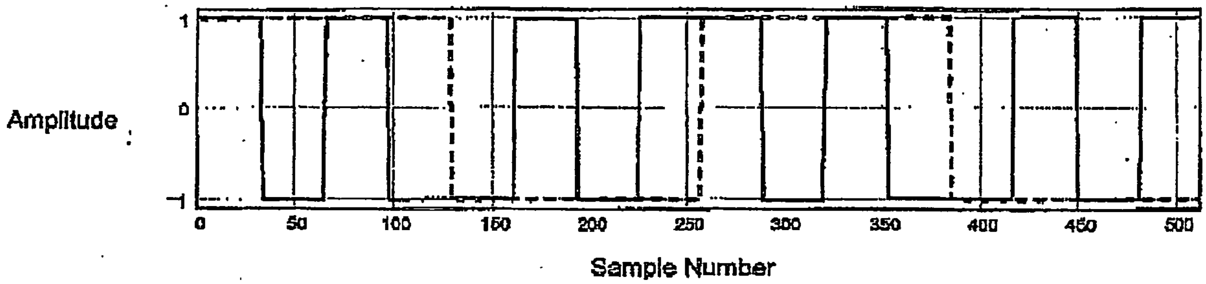


Figure 6 – Double Binary Offset Carrier Modulatio
total duration equal to A in figure 2.

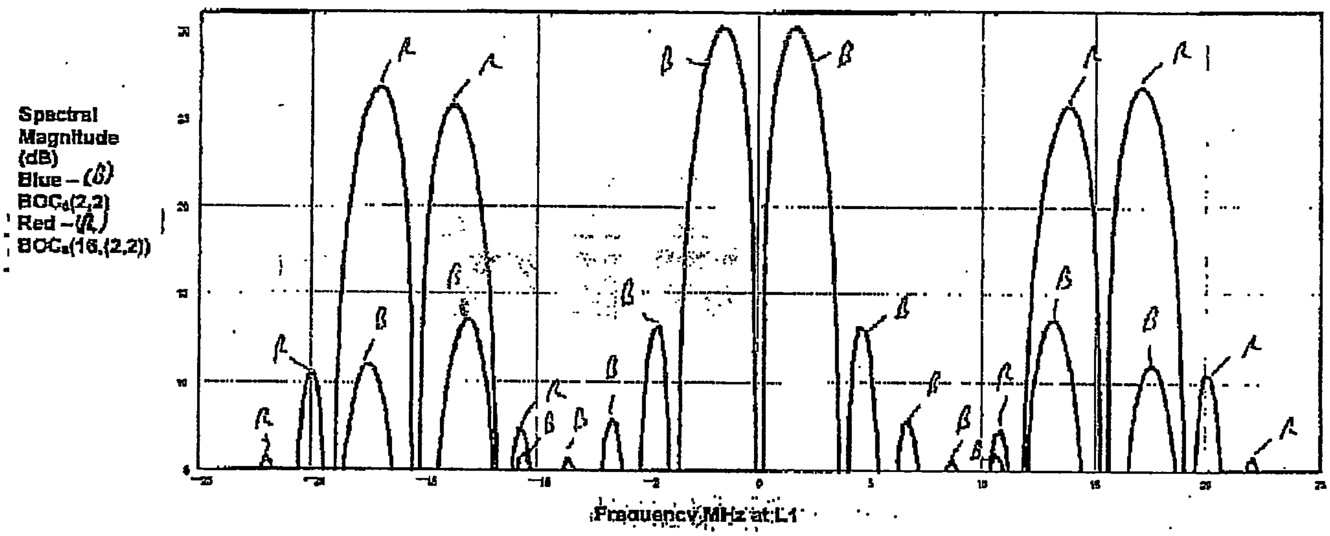
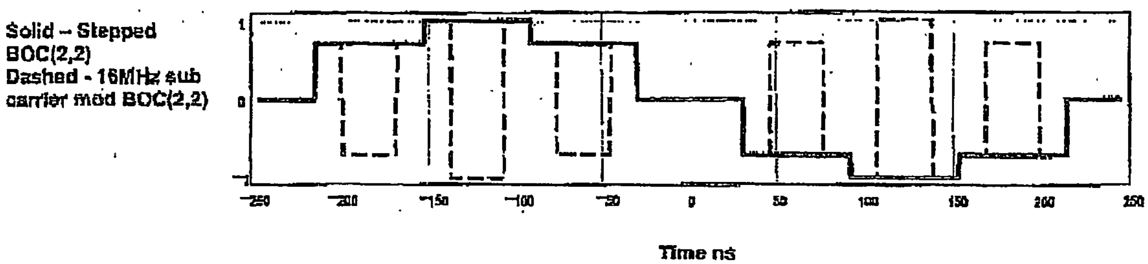


Figure 7 – Example of a Double BOC Spectrum, DBOC(16,(2,2)), comparison with BOC(2,2

16MHz sub carrier modulated BOC(2,2) wave



Time shifted BOC(2,2) wave – phase quadrature with BOC(16,(2,2))

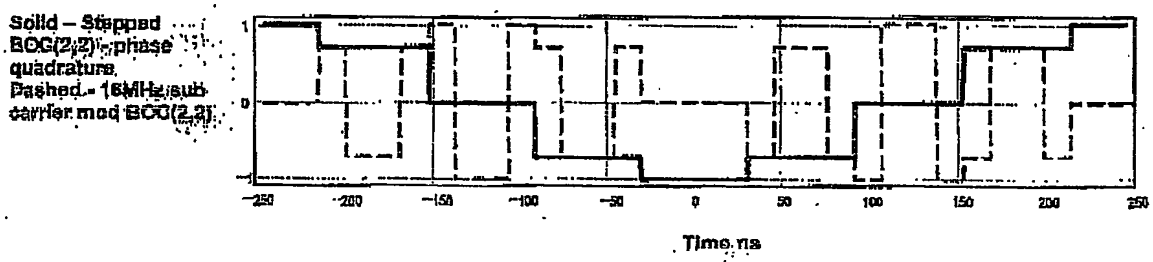


Figure 8 – Stepped sine wave DBOC Modulation for I channel and BOC Modulation for Q channel exhibiting constant carrier magnitude

The reader is additionally referred to the following paper (BOC Modulation Waveforms) by the inventors giving further information and explanation.

BOC Modulation Waveforms

Anthony R Pratt, *Orbstar Consultants*, UK and John I R Owen, *DSTL, MOD, UK*

ABSTRACT

This paper is concerned with the evolution of modulation waveforms that are starting to find application in the most recent navigation satellite transmissions. Specifically, we focus on the waveforms used for sub carrier modulation with binary signals, the spectral consequences of the resulting waveforms and the generalization of these signals. Specifically, we consider multi-level waveforms that can be simply implemented by multi-phase constant envelope signals, well adapted to high power amplifiers used in navigation satellite payloads. The method of superposition is used to derive a family of equations describing the spectra of such signals. Formulae are derived for the spectra of both 3-level and 5-level BOC sub carrier waveforms. The theory is general and may be easily extended to both sub carrier modulation with higher numbers of levels and the use of non-binary spreading code waveforms.

One important suggestion, which arose from J-L Issler (CNES), is the use of cosine sub carrier modulation, rather than sine wave modulation. The results of analysis are also applied to this case.

One of the metrics by which the new signals are compared for their interactions with others is the SSC. Consequently, a section is included in the paper setting out a more developed form of the Spectral Separation Coefficient Theory. There are two useful enhancements to the theory. One permits the introduction of protection filters to assist in the isolation of one signal type from another whilst the second derives a form that is computable in the time domain – this is especially beneficial for complicated signal shapes.

The combination of the two is a more complete theory in providing an armory of techniques useful in designing navigation signals able to reasonably compete within the current crowded spectra. This especially useful in the design of candidate signals for Galileo, able to occupy the E1-L1-E2 frequency band alongside CA code transmissions from GPS and the BOC(10,5) spectrum proposed for M-code.

Biography

Dr Tony Pratt graduated with a B.Sc. and Ph.D. in Electrical and Electronic Engineering from Birmingham University, UK. He joined the teaching staff at Loughborough University, UK in 1967 and remained until 1980. He held visiting professorships at Yale University, IIT, New Delhi and University of Copenhagen. In 1980, he joined Navstar Ltd, as Technical Director. He was involved in the formation of Tollstar Ltd, a 5 company consortium pursuing the development of Electronic Road Tolling, running this company until 1996. He is now Technical Director (GPS) with Parthus, Special Professor at the IESSG at the University of Nottingham, UK. He is a Consultant to the UK Government in the development of Galileo Satellite System.

John Owen is Team Leader, Navigation Systems, Air Systems Department, Defence Science and Technology Laboratory, Dstl. He is a Dstl Fellow, a Fellow of the Royal Institute of Navigation, a Fellow of the Intuition of Electrical Engineers and a Chartered Engineer. After gaining an Honours Degree in Electrical and Electronic Engineering at Loughborugh University of Technology he joined the UK MOD's research establishment. His association with satellite navigation research began in the late 1970's. During the 1980's he was responsible for the research contracts that developed digital GPS receivers, anti jamming antenna systems and signal simulators. He is technical adviser to MOD for their Navigation Warfare programme and to UK Government Departments for the European Galileo programme.

INTRODUCTION

Considerable efforts have been expended in the development and qualification of signals for the Galileo satellite navigation system. These have reached baseline status is all of the proposed transmission bands (E5, E6, E1-L1-E2) but concerns remain in the E1-L1-E2 band because this is already occupied with GPS transmissions. Levels of intra-system and inter-system interference have been computed by a number of agencies in Europe and the USA. The additional interference generated by Galileo is not considered to be at a level which significantly degrades other satellite navigation systems and is well inside ITU measures for the control of interference within geo-stationary satellite systems. This not the only measure of interaction between the signals and receivers of 2 satellite systems and issues of jamming need also to be considered. This is especially appropriate for Galileo where it is intended that the system include 'local elements' – some of which may take the form of ground or near ground transmitters with identical signal formats to those of the satellites.

This has motivated the search for improved satellite waveforms exhibiting lower levels of interference with existing systems whilst still providing good performance. However, the main metric for establishing the interaction between the signals from one satellite system (or service as established by spectrum) and the receiver performance for another satellite system or different service within the same system, is the Spectral Separation Coefficient. This seems to have been introduced originally by Betz (ref 1, 2) but there may have been earlier origins. He used it to deduce performance degradations in received signal to noise levels.

We generalise the concept to be a measure of the noise power output from a receiver when certain signals with given spectra are incident in its input. This shows that the fundamental measure is a cross power spectral density. This has particular utility in designing signal structures with good opportunities for co-existence within a given frequency band. From this idea, we further introduce the concept of Partial Spectral Separation Coefficients in the which the frequency bands for the accumulation of noise energy are split into disjoint regions (non-disjoint regions or even overlapping regions could be used) as a diagnostic tool to establish numerically the relative contributions of each sub-band. Finally, in the area of SSC, we introduce a new means to compute SSC's in the time domain using a 'cross autocorrelation' function. This term does not seem to have appeared previously in the literature and will be defined as the Fourier Transform of the cross power spectral density. The time domain method of SSC computation has two advantages. Firstly, it computes the SSC over an infinite frequency spectrum. This, it is argued, is the correct normalization for SSC's. Secondly, when the modulation waveforms are complicated, it provides a much simpler method of computation.

Finally, the theory of SSC's is evolved to include the effects of filters prior to the main signal processing in satellite navigation receivers. This one of the newer techniques which can be employed to separate the effects of satellite signals with different spectra. The filters improve the effective Spectral Separation Coefficients (broadly read as performance in jamming) but at the (slight) cost of sub-optimum system performance against a white noise background.

SPECTRAL SEPARATION COEFFICIENTS

Spectral Separation Coefficients represent the power at the output of a receiver matched filter when subject to certain input signals. The general arrangement is shown in figure 1.1. The input to the receiver might be from a variety of possible sources as shown. These include a signal source whose spectrum ($H_S(\omega)$), is matched to that of the receiver (in the signal processing sense – this specifies that the receiver filter is the complex conjugate of the signal spectrum - $H_S^*(\omega)$) or an interfering signal with spectrum $H_I(\omega)$. The figure also has a 'protection' filter between the signal inputs and the receiver matched filter. This can be considered as having a frequency transfer response of 1 everywhere when it is not required.

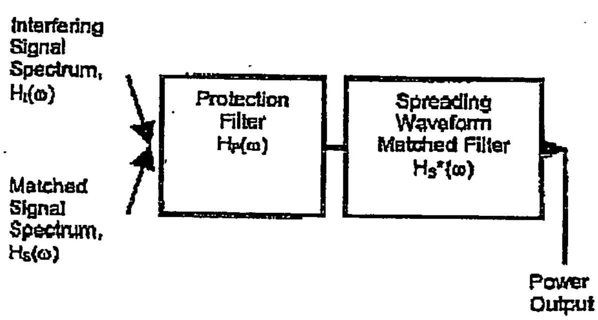


Figure 1.1 – Arrangement of signals incident on protected receiver

The spectrum at the output of the (spreading waveform) matched filter is:

$$S_O(\omega) = (P_I^{1/2}.H_I(\omega) + P_S^{1/2}.H_S(\omega)).H_P(\omega).H_S^*(\omega) \tag{1.1}$$

Note that the definitions of $H_I(.)$ and $H_S(.)$ are general and could encompass also the spreading codes as well as the spreading waveform. This allows for a further development of the SCC theory including the complex code signals.

The P_I and P_S multipliers represent the power levels (signal levels) of the two sources respectively. The power output is just the integral (over frequency) of this spectrum, assuming that the protection filter is absent:

$$P_O(\omega) = \{P_S|H_S(\omega)|^2 + P_I|H_I(\omega)|^2\}|H_S(\omega)|^2 + 2P_I^{1/2}.P_S^{1/2}.H_S(\omega).H_I(\omega)|H_S(\omega)|^2 \quad (1.2)$$
$$= \{P_S.\Phi_S(\omega) + P_I.\Phi_I(\omega)\}|H_S(\omega)|^2$$

The step from equations 1.1 to 1.2 involving the squaring of the sum of the 2 signal spectra induces a cross product mathematically of the form $H_S().H_I()$. This component disappears in the following integrals since it is assumed that the two spectra arise from signals that are uncorrelated. This effect is due to either the code structure of the satellite signals even if the spectral envelopes are identical or from the fact that the spectra share no common components. In other words the cross-spectrum term averages to zero. This is an important assumption implicitly used in the application of SSC's and may not be satisfied in a number of important cases. One example is to be found in intra-system effects where the cross-correlation effects between finite length codes is likely to be an important contributor. However, the term is dropped in the second line of equation 1.2 for convenience.

The terms, Φ, are the power spectral densities of the two spectra. This is the correct designation since the definitions above have the integrated power in each spectrum is:

$$P_I = \int_{-\infty}^{\infty}|P_I^{1/2}.H_I(\omega)|^2.df \quad \text{since}$$
$$\int_{-\infty}^{\infty}|H_I(\omega)|^2.df = 1 \quad \text{and by Parseval's Theorem}$$
$$\phi_I(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}|H_I(\omega)|^2.\exp(j\omega t).d\omega \quad \text{(Fourier Transform)}$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_I(\omega).\exp(j\omega t).d\omega \quad \text{from which at } t = 0$$
$$\phi_I(0) = 1.0$$

(1.3)

Similar relationships exist also for the signal spectrum, defined as having unity power. Consequently, the power output from the matched filter is:

$$\bar{P}_O = \int P_O(\omega).df$$
$$= \int_{-\infty}^{\infty} P_I.\Phi_I(\omega)|H_S(\omega)|^2.df + \int_{-\infty}^{\infty} P_S.\Phi_S(\omega)|H_S(\omega)|^2.df$$
$$= P_I.\int_{-\infty}^{\infty}\Phi_I(\omega).\Phi_S(\omega).df + P_S.\int_{-\infty}^{\infty}\Phi_S^2(\omega).df$$

(1.4)

The spectral separation coefficient of the a signal with itself, with power spectral density $\Phi_S(\omega)$ is defined as:

$$\kappa_{SS} = \int_{-\infty}^{\infty}\Phi_S^2(\omega).df = 2.\int_0^{\infty}\Phi_S^2(\omega).df \quad (1.5)$$

The spectral separation coefficient between signals with different spectra, such as $\Phi_I(\omega)$ and $\Phi_S(\omega)$, is defined as

$$\kappa_{IS} = \int_{-\infty}^{\infty}\Phi_I(\omega).\Phi_S(\omega).df \quad (1.6)$$

Equations 1.5 and 1.6 assume that the receiver and transmitter have infinite bandwidth. If this is not the case (as is usual in practice), either the integration limits require adjustment or corrections to the SSC's must be made to include the effects of the finite bandwidths of receiver and transmitter.

When a (protective) filter is inserted in the signal path between the satellite signal (generator) and the receiver matched filter, the spectral separation coefficients are changed for any given pair of signals:

$$\hat{P}_O = \int P_O(\omega).df$$

$$= \int_{-\infty}^{\infty} P_I.\Phi_I(\omega).\left|H_P(\omega).H_S(\omega)\right|^2.df$$
$$+ \int_{-\infty}^{\infty} P_S.\Phi_S(\omega).\left|H_P(\omega).H_S(\omega)\right|^2.df \qquad (1.7)$$
$$= P_I.\int_{-\infty}^{\infty} \Phi_I(\omega).\left|H_P(\omega)\right|^2.\Phi_S(\omega).df$$
$$+ P_S.\int_{-\infty}^{\infty} \Phi_S^2(\omega).\left|H_P(\omega)\right|^2.df$$

and the revised spectral separation coefficients become:

$$\kappa_{IPS} = \int_{-\infty}^{\infty} \Phi_I(\omega).\left|H_P(\omega)\right|^2.\Phi_s(\omega).df$$
$$\kappa_{SPS} = \int_{-\infty}^{\infty} \left|H_P(\omega)\right|^2.\Phi_s^2(\omega).df \qquad (1.8)$$

The spectral separation coefficients may be significantly reduced by the action of the 'protective' filter in reducing the magnitude of those parts of the spectrum in which there are significant cross spectral components – $\{H_I(\omega).H_S(\omega)\}$, (signal interactions).

Cross Power Spectra

In this section, we explore various relationships between the components involved in spectral separation coefficients as these generate additional insight and new means of performing the required calculations. These will bring computational benefits under certain conditions.

We note that there is a fundamental element in the SSC which we call the cross power spectrum, $\Phi_{12}(\omega)$:

$$\Phi_{12}(\omega) = \Phi_1(\omega).\Phi_2(\omega) \qquad (2.1)$$

The cross power spectrum is fundamental since SSC's and partial SSC's can be derived from it by integration. By examining the variation of $\Phi_{12}(\omega)$, it can be observed where the main interactions between the two spectra lie in the frequency domain. This is an important diagnostic tool in establishing the efficacy or otherwise of protective filters or other spectrum controlling affects. An example of the cross power spectrum of BOC(2,2) and BOC(10,5) signals is given in figure XX.

To continue the analysis, each of the component power spectral densities is replaced by the Fourier Transform of the related auto-correlation function:

$$\Phi_{12}(\omega) = \int_{-\infty}^{\infty} \phi_1(\tau).\exp(-j\omega\tau).d\tau.\int_{-\infty}^{\infty} \phi_2(\varsigma).\exp(-j\omega\varsigma).d\varsigma$$
$$= \iint \phi_1(\tau).\phi_2(\varsigma).\exp(-j\omega(\tau+\varsigma)).d\tau d\varsigma \quad \text{and then}$$
$$\phi_{12}(\upsilon) = \frac{1}{2\pi}\iiint \phi_1(\tau).\phi_2(\varsigma).\exp(-j\omega(\tau+\varsigma-\upsilon))d\omega d\tau d\varsigma$$
$$(2.2)$$

The last step takes the Fourier Transform of the cross power spectrum to form a cross auto-correlation function. By altering the order of the integrations, performing the one in ω first, we find:

$$\phi_{12}(\upsilon) = \iint \phi_1(\tau).\phi_2(\varsigma).\delta(\upsilon-\tau-\varsigma).d\tau d\varsigma$$
$$= \int_{-\infty}^{\infty} \phi_1(\tau).\phi_2(\upsilon-\tau).d\tau \qquad (2.3)$$

The last result comes from the integration with respect to ζ, recognising a form of the Dirac δ-function. The result is not surprising – it is simply that the cross auto-correlation function is the convolution of the two signal auto-correlation functions. The is useful because, by applying the Weiner-Khinchine Theorem, the spectral separation coefficient can be easily deduced from the signal auto-correlation functions:

$$\phi_{12}(\upsilon) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{12}(\omega)\exp(j\omega\upsilon).d\omega$$
$$\kappa_{12} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{12}(\omega).d\omega$$
$$= \phi_{12}(0) \qquad (2.4)$$
$$= \int_{-\infty}^{\infty} \phi_1(\tau).\phi_2(-\tau).d\tau$$
$$= \int_{-\infty}^{\infty} \phi_1(\tau).\phi_2(\tau).d\tau$$

The last step arises since auto-correlation functions always even functions of time (the power spectral densities are always real functions). Consequently, the SSC of a signal with itself is:

$$\kappa_{11} = \int_{-\infty}^{\infty} \phi_{11}^2(\tau).d\tau \qquad (2.5)$$

Protective Filters

The results of the previous section have been derived without the provision of the protective filter.

In this section, these addition results are deduced from the equations above. The easy path to the necessary results is from equation 1.8 from which it can be seen that the spectral separation equations include an extra term – the power frequency response of the filter. In what follows we write:

$$\Phi_P(\omega) = \Phi_3(\omega) = \left|H_P(\omega)^2\right| \tag{3.1}$$

Then the cross power spectral density for a filter receiver combination becomes after equation 2.5:

$$\Phi_{123}(\omega) = \Phi_1(\omega).\Phi_2(\omega).\Phi_3(\omega) \tag{3.2}$$

Apply a (reversible) Fourier Transform to each of the component power spectral densities yields:

$$\Phi_{123}(\omega) = \iiint \phi_1(\tau).\phi_2(\varsigma).\phi_3(\eta).\exp(-j\omega(\tau+\varsigma+\eta))d\tau d\varsigma d\eta$$

and

$$\phi_{123}(\upsilon) = \frac{1}{2\pi}\iint\iint \begin{Bmatrix} \phi_1(\tau).\phi_2(\varsigma).\phi_3(\eta). \\ \exp(-j\omega(\tau+\varsigma+\eta-\upsilon)) \end{Bmatrix} d\omega d\tau d\varsigma d\eta \tag{3.3}$$

This is similar to equation 2.2. By applying the inverse transform to the ω term, the equation becomes the triple convolution of the 3 autocorrelation functions (the filter 'auto-correlation function being the inverse transform of the power frequency response). Consequently, the SSC for signal pairs that are subject to a filtering process are:

$$\kappa_{123} = \int_{-\infty}^{\infty} \phi_1(\tau).\phi_2(\tau).\phi_3(\tau).d\tau$$
$$\kappa_{131} = \int_{-\infty}^{\infty} \phi_1^{\,2}(\tau).\phi_3(\tau).d\tau \tag{3.4}$$

Structural Properties Of SSC's

Considerations of the means to perform numerical analysis of the integral in equation 2.5 and 3.4 lead to the identification of 2 components – a duration component and a shape component. The reason for this division is simply the finite duration of the non-zero portion of the integrals in equations 2.4 and 2.5. These can never be larger than twice the duration of the shortest signal considered. Most often, this will be the spreading waveform itself.

For example, the CA code (BPSK-R1) signal has an autocorrelation function extending over (2/1.023)µs, giving a duration component of –57.08dBs. The shape component (of the triangular autocorrelation function spread over a unit of time measurement) is found to be –4.77dBs. This makes the overall self SSC for a CA code signal equal to –61.86dBs. A BPSK-R10 signal (for example, P(Y) code) has an identical shape factor but a duration component of –67.08dBs, giving an overall κ_{11} of –71.86dBs.

When several different signals are used, the duration of the autocorrelation of the shortest signal determines the non-zero region for the integrand in 2.4 and 2.5. However, whilst the duration component is easy to determine, the shape component may be more complicated.

This completes the section on the development of spectral separation coefficients necessary to perform the analysis on advanced BOC waveforms.

BOC MODULATION WAVEFORMS

We will start the section on BOC modulation waveforms and their spectra by revisiting the spectrum of the (well known) BPSK modulation with a rectangular spreading wave. This will serve two purposes – to establish the nomenclature and to touch upon the assumptions necessary to support the current use of the (average) envelope of the actual spectra and SSC's.

BPSK-R Modulation

We start with the spectrum of a version of the CA code, $\{c_r\}$, in which the spreading code sequence is infinitely long with 2 equi-probable states, c_r. The auto-correlation function for the spreading code sequence is defined as unity at zero time offset and zero elsewhere – a perfect random code.

$$\phi(j) = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{r=-N}^{N} c_r.c_{r+j}$$
$$\phi(j) = 1 \quad \text{for } j = 0$$
$$= 0 \quad \text{for } j \neq 0 \tag{5.1}$$

The spreading signal, s(t), is the convolution of the code sequence with a spreading waveform, h(t)

$$h(t) \quad \text{where } -\infty < t < +\infty \tag{5.2}$$

then

$$s(t) = \sum_{r=-\infty}^{+\infty} c_r.h(t - r.\Delta T). \tag{5.3}$$

where the spreading code, c_r, takes a new state every ΔT seconds.
A typical (rectangular) spreading waveform, h(t), for a BPSK-R modulation is:

$$h(t) = 1 \quad \text{for } |t| \leq \Delta T/2, \text{ and}$$
$$= 0 \quad \text{elsewhere} \tag{5.4}$$

The spectrum, H(ω), is the Fourier Transform of h(t) - the well known sinc function:

$$H(\omega) = \Delta T . \frac{\sin(\omega\Delta T/2)}{(\omega\Delta T/2)} \qquad (5.5)$$

The power spectral density, $\Phi_s(\omega)$, for the waveform s(t) is the Fourier Transform of its autocorrelation function (Wiener-Khintchine Theorem). The computation of $\Phi_s(\omega)$ is probably best performed via the autocorrelation function since the spectrum of the sequence $\{c_r\}$ is not well defined in terms of its phase profile. Otherwise, use could be made of the convolution to product transformation which occurs via the Fourier Transform.

$$\Phi_s(\omega) = \int_{-\infty}^{\infty} \phi_s(\tau).\exp(-j\omega\tau).d\tau$$
$$\phi_s(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_s(\omega).\exp(j\omega\tau).d\omega \qquad (5.6)$$

The auto-correlation function of s(t) is:

$$\phi_s(\tau) = \int_{-\infty}^{\infty} s(t).s(t-\tau).dt$$
$$= \int_{-\infty}^{\infty} \sum_r \sum_n c_r c_n .h(t - r\Delta T).h(t - \tau - n\Delta T).d\tau \qquad (5.7)$$

Since, at least, h(u) is non-zero only over the range $|u| < \Delta T$, the integral and summations can be easily simplified. Additionally, the product $[c_r c_n]$ is only non-zero on average in the region r=n. Hence:

$$\phi_s(\tau) = 1 - \left|\frac{\tau}{\Delta T}\right| \quad \text{for } |\tau| < \Delta T$$
$$= 0 \quad \text{elsewhere} \qquad (5.8)$$

and

$$\Phi_s(\omega) = H(\omega).H^*(\omega)$$
$$= \Delta T . \frac{\sin^2(\omega.\Delta T/2)}{(\omega\Delta T/2)^2} \qquad (5.9)$$

$H^*(\omega)$ is the complex conjugate of H(ω). Using the Wiener-Khintchine Theorem, the area under the function, $\Phi_s(\omega)$ with respect to the frequency variable f, is unity since:

$$\phi_s(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_s(\omega).\exp(j\omega\tau).d\omega$$
$$\phi_s(0) = \int_{-\infty}^{\infty} \Phi_s(\omega).\exp(j\omega.0).df \qquad (5.11)$$
$$= 1$$

Note should be made here of the relationship between $|H(\omega)|^2$ and $\Phi_s(\omega)$. This is a factor of $1/\Delta T$, which is the power spectral density of the random code signal (see for example ref 3).

BOC Modulation

In the case of BOC modulation, the rectangular spreading waveform of equation 5.4 [h(t)], is further modulated by a sub-carrier having an integer number of half cycles within the duration ΔT. If the sub-carrier modulation is sinusoidal, the modulation is known as LOC (linear offset carrier) whilst if the sub-carrier modulation is rectangular, the modulation is known as BOC (binary offset carrier).

There are other possibilities for the sub-carrier modulation also, for example, using stepped approximations to a sinusoid. Some of these may be useful in satellite implementations. For this reason, the initial analysis will be focused on a 3-level stepped waveform for the sub-carrier modulation. The levels used are (-1, 0, +1) and the times of transition between the levels will be changeable (at least to some degree). This is shown in figure XY. This analysis method has sufficient flexibility to encompass BOC modulation (binary), tertiary and 5-level stepped waveforms by means of superposition.

We model the offset carrier by revising the definition of the spreading waveform to $h_{OC}(t)$ through multiplication of the basic spreading waveform with the sub-carrier waveform:

$$h_{OC}(t) = h(t).c_{SC}(t)$$
$$\text{where } c_{SC}(t) \text{ is the sub-carrier waveform} \qquad (6.1)$$

The spectrum of a repetitive binary waveform is well known (whether $c_{SC}(t)$ has odd or even symmetry about t=0 depends on both the phasing of the sub-carrier and whether m is even or odd), but we will not use this technique. We set up the specific waveform for the non-zero interval of h(t), containing m half cycles of the sub-carrier. There are several cases. The first two (cases 1 & 2) are concerned with a sub carrier modulation which is a digital version of a sine wave in phasing with respect to the spreading waveform. The next 2 cases (3 & 4) use a cosine phasing for the sub carrier modulation.

Case 1: sine phased sub carrier waveform with m even. It does not matter whether there is a positive or negative transition at t=0 since this only causes a 180 degree phase shift in the fundamental component, translated into all the harmonics. The definition of the 3-level BOC modulated spreading waveform is specified during each ΔT/m interval:

$$c_{SC}(t)=0 \qquad r.\frac{\Delta T}{m}<t<(r+\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=(-1)^r \qquad (r+\delta).\frac{\Delta T}{m}<t<(r+1-\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=0 \qquad (r+1-\delta).\frac{\Delta T}{m}<t<(r+1).\frac{\Delta T}{m}$$
$$\text{for } r \text{ in } -m/2..(+m/2-1)$$

(6.2)

This representation has a negative transition at t=0 and the waveform has dwells at the 0 level, between each +ve or -ve state, of duration (2δ.ΔT/m) in comparison with the dwell at level 1 of ((1-2δ).ΔT/m). The BOC waveform (binary sub carrier) has δ=0.

Case 2: sine phased sub carrier waveform with m odd. This implies a positive or negative state at t=0. The results are phase changed by 180 degrees only. The 3 level stepped sub carrier waveform, with dwells at zero of (2δ.ΔT/m), is:

$$c_{SC}(t)=0 \qquad (r-1/2).\frac{\Delta T}{m}<t<(r-1/2+\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=(-1)^r \qquad (r-1/2+\delta).\frac{\Delta T}{m}<t<(r+1/2-\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=0 \qquad (r+1/2-\delta).\frac{\Delta T}{m}<t<(r+1/2).\frac{\Delta T}{m}$$
$$\text{for integer } r \text{ in the range } -(m-1)/2..+(m-1)/2 \qquad (6.3)$$

Since m is odd, (m-1) is even and can be divided by 2, resulting in integer values. This equation defines a positive state for the BOC modulation at t=0. The overall limits for the sub carrier modulated spreading waveform are ±m.ΔT/2.

Case 3: cosine phased sub carrier with m even. The same waveform synthesis techniques are applied as in the previous cases. The 3 level stepped sub carrier waveform with dwells of (2δ.ΔT/m), is:

$$c_{SC}(t)=(-1)^r \qquad r.\frac{\Delta T}{m}<t<(r+1/2-\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=0 \qquad (r+1/2-\delta).\frac{\Delta T}{m}<t<(r+1/2+\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=-(-1)^r \qquad (r+1/2+\delta).\frac{\Delta T}{m}<t<(r+1).\frac{\Delta T}{m}$$
$$\text{for integer } r \text{ in } -m/2..+(m/2-1) \qquad (6.4)$$

Case 4: cosine phased sub carrier waveform with m odd. This implies a positive or negative state at t=0. The results are phase changed by 180 degrees only. The 3 level stepped sub carrier waveform, with dwells at zero of (2δ.ΔT/m), is:

$$c_{SC}(t)=(-1)^r \qquad (r-1/2).\frac{\Delta T}{m}<t<(r-\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=0 \qquad (r-\delta).\frac{\Delta T}{m}<t<(r+\delta).\frac{\Delta T}{m}$$
$$c_{SC}(t)=-(-1)^r \qquad (r+\delta).\frac{\Delta T}{m}<t<(r+1/2).\frac{\Delta T}{m}$$
$$\text{for integer } r \text{ in } -(m-1)/2..+(m-1)/2 \qquad (6.5)$$

Since m is odd, (m-1) is even and can be divided by 2, resulting in integer values. This equation defines a positive state for the BOC modulation at t=0. The overall limits for the sub carrier modulated spreading waveform are ±m.ΔT/2.

Equations 6.2 to 6.5, set up repetitive tertiary waveforms, with levels (-1, 0, +1), with transitions at intervals of (ΔT/m) and dwells at zero of duration (2δ.ΔT/m) each. This arrangement provides for m half cycles of the binary sub-carrier during the non-zero interval of the spreading waveform, h(t).

It is clear that other waveforms can be made from this 'building block' by superposition and specification of δ. This allows construction of the sub-carrier waveform including (but not limited to) 2, 3 or 5 level stepped waveforms. These are particularly interesting as they could be implemented with minimal extra complexity and offer real performance gains. The effects can be computed through the spectrum of the sub-carrier.

In performing the analysis for this waveform component at this stage, it is possible to avoid repeating the analysis for each waveform type later.

Case 1: The Fourier Transform of $c_{SC}(t)$ for m even, is:

$$C_{SC}(\omega)=\int_{-\infty}^{\infty} c_{SC}(t).\exp(-j\omega t).dt$$
$$=\sum_{r=-m/2+1/2}^{+(m/2-1/2)}\left\{\int_{t=(r-\frac{1}{2}+\delta)\Delta T/m}^{(r+\frac{1}{2}-\delta)\Delta T/m}(-1)^r.\exp(-j\omega t).dt\right\}$$
$$=\frac{2}{\omega}.\sin(\omega.\frac{\Delta T}{2m}(1-2\delta)).\left\{\sum_r(-1)^r\exp(-j\omega r\frac{\Delta T}{m})\right\}\qquad(6.6)$$

The final step in evaluating 6.6 concentrates on the summation. This can easily be performed using the infinite series derived from 1/(1-x). We define:-

$$x=(-1).\exp(-j\omega\frac{\Delta T}{m})\qquad(6.7)$$

Then, the summation in 6.6 is denoted as U (with the limits as set therein):

$$U=\sum_r(-1)^r.\exp(-j\omega r\frac{\Delta T}{m})$$
$$=\sum_{r=-m/2+1/2}^{+m/2-1/2}x^r$$
$$=\left\{\frac{x^{-m/2}-x^{+m/2}}{x^{-1/2}-x^{1/2}}\right\}\qquad(6.8)$$
$$=-(-1)^{-m/2}\left\{\sin(\omega\frac{\Delta T}{2})/\cos(\omega\frac{\Delta T}{2m})\right\}$$

In the last step, the sine function in the numerator is present since m is even and $(-1)^m$ is always +1. Then

$$C_{SC}(\omega)=\frac{-2.(-1)^{-m/2}}{\omega}.\sin(\omega.\frac{\Delta T}{2m}(1-2\delta)).\left\{\sin(\omega.\frac{\Delta T}{2})/\cos(\omega.\frac{\Delta T}{2m})\right\}\qquad(6.9)$$

We note that the frequencies of the sub carrier and chipping rates can be denoted as f_m, f_c, where:

$$f_m=\frac{m}{2\Delta T}\quad\text{or}$$
$$\omega_m=\frac{\pi.m}{\Delta T}\qquad(6.10)$$
$$f_c=\frac{1}{\Delta T}$$

The power spectral density is formed from equation 6.6 by multiplying by its complex conjugate:

$$|C_{SC}(\omega)|^2=(\Delta T)^2.\left\{\frac{\sin(\omega.\frac{\Delta T}{2m}(1-2\delta)).\sin(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2\qquad(6.11)$$

Consequently, the power spectral density for the BOC signal including the code waveform is:

$$\Phi_{BOCs}(\omega)=\frac{\Delta T}{(1-2\delta)}.\left\{\frac{\sin(\omega.\frac{\Delta T}{2m}(1-2\delta)).\sin(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2\qquad(6.12)$$

It is worthy to note that the spectrum can be modified by the presence of the factor (1-2δ) in one of the sine functions in the numerator. The power spectral density for the BOC modulation (with δ=0) is:

$$\Phi_{BOC}(\omega)=\Delta T.\left\{\frac{\sin(\omega.\frac{\Delta T}{2m}).\sin(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2\qquad(6.13)$$

This corresponds exactly with Betz (ref 1). This is plotted for a BOC(2,2) and BOC(10,5) in figure 3.

Case 2: m odd. The formulation proceeds similarly from the previous case (m even), but with different limits, until the expressions start to simplify:

$$C_{SC}(\omega)=\int_{-\infty}^{\infty} c_{SC}(t).\exp(-j\omega t).dt$$
$$=\sum_{r=-(m-1)/2}^{+(m-1)/2}\left\{\int_{t=(r-\frac{1}{2}+\delta)\Delta T/m}^{(r+\frac{1}{2}-\delta)\Delta T/m}(-1)^r.\exp(-j\omega t).dt\right\}$$
$$=\frac{2}{\omega}.\sin(\omega.\frac{\Delta T}{2m}(1-2\delta)).\left\{\sum_r(-1)^r\exp(-j\omega r\frac{\Delta T}{m})\right\}\qquad(6.14)$$

This is very nearly the same expression as for the case of m even. The evaluation of the summation can proceed in exactly the same way via the infinite series 1/(1-x) as before with the definition of x as equation 6.7:

$$U=\sum_{r=-(m-1)/2}^{+(m-1)/2}(-1)^r.\exp(-j\omega\frac{\Delta T}{m})$$
$$=(-1)^{-(m-1)/2}.\left\{\frac{\cos(\omega\frac{\Delta T}{2})}{\cos(\omega\frac{\Delta T}{2m})}\right\}\qquad(6.15)$$

The numerator of the expression in equation 6.12 is a sine function because the $(-1)^{(m+1)}$ multiplier of the second term is always positive due to the fact that m is odd.

The power spectral density for the BOC modulated spreading waveform is, therefore:

$$|C_{SC}(\omega)|^2 = \{\Delta T\}^2 . \left\{ \frac{\sin(\omega.\frac{\Delta T}{2m}(1-2\delta)).\cos\left\{\omega.\frac{\Delta T}{2}\right\}}{(\omega.\frac{\Delta T}{2}).\cos\left\{\omega.\frac{\Delta T}{2m}\right\}} \right\}^2 \tag{6.16}$$

This expression corresponds that of Betz (equation A12 in reference 1) with $\delta = 0$. Finally, the power spectral density for the code signal modulated by a δ BOC spreading signal is:

$$\Phi_{BOC3}(\omega) = \frac{\Delta T}{(1-2\delta)} . \left\{ \frac{\sin(\omega.\frac{\Delta T}{2m}(1-2\delta)).\cos\left\{\omega.\frac{\Delta T}{2}\right\}}{(\omega.\frac{\Delta T}{2}).\cos\left\{\omega.\frac{\Delta T}{2m}\right\}} \right\}^2 \tag{6.17}$$

The power spectral density for a binary sub carrier modulation (with δ=0) is:

$$\Phi_{BOC}(\omega) = \Delta T . \left\{ \frac{\sin(\omega.\frac{\Delta T}{2m}).\cos\left\{\omega.\frac{\Delta T}{2}\right\}}{(\omega.\frac{\Delta T}{2}).\cos\left\{\omega.\frac{\Delta T}{2m}\right\}} \right\}^2 \tag{6.18}$$

Case 3: The Fourier Transform of $c_{SC}(t)$ for m even with a cosine sub carrier as defined above in equation 6.4, is:

$$\begin{aligned} C_{SC} &= \int_{-\infty}^{\infty} c_{SC}(t).\exp(-j\omega t).dt \\ &= \sum_{r=-m/2}^{+(m/2-1)} \left[\begin{aligned} &\int_{t=r.\Delta T/m}^{(r+1/2-\delta).\Delta T/m} (-1)^r.\exp(-j\omega t).dt \\ &+ \int_{t=(r+1/2+\delta)\Delta T/m}^{(r+1)\Delta T/m} -(-1)^r.\exp(-j\omega t).dt \end{aligned} \right] \\ &= \frac{-2}{j\omega}.\exp(-j\omega.\frac{\Delta T}{2m}). \left[\begin{aligned} &\left\{\cos(\omega.\delta\frac{\Delta T}{m}) - \cos(\omega.\frac{\Delta T}{2m})\right\} \\ &\left\{\sum_r (-1)^r \exp(-j\omega r\frac{\Delta T}{m})\right\} \end{aligned} \right] \end{aligned} \tag{6.19}$$

The last step in equation 6.19 moved all non r-related components from the summation recognising two cosine components. The final step in evaluating equation A2.19 concentrates on the summation which can be performed as before. We define:

$$x = (-1).\exp(-j\omega\frac{\Delta T}{m}) \tag{6.20}$$

Then, the summation is denoted as U:

$$\begin{aligned} U &= \sum_{r=-m/2}^{m/2-1} (-1)^r.\exp(-j\omega r\frac{\Delta T}{m}) \\ &= x^{-1/2}\left\{\frac{x^{-m/2} - x^{+m/2}}{x^{-1/2} - x^{+1/2}}\right\} \\ &= -j.(-1)^{-m/2}.\exp(j\omega\frac{\Delta T}{2m}).\left\{\sin(\omega\frac{\Delta T}{2}) / \cos(\omega\frac{\Delta T}{2m})\right\} \end{aligned} \tag{6.21}$$

Then:

$$C_{SC} = \frac{-2.(-1)^{-m/2}}{\omega}.\left\{\cos(\omega\delta.\frac{\Delta T}{m}) - \cos(\omega.\frac{\Delta T}{2m})\right\}.\left\{\frac{\sin(\omega.\frac{\Delta T}{2})}{\cos(\omega.\frac{\Delta T}{2m})}\right\} \qquad (6.22)$$

The power spectral density is formed from equation 6.22 by multiplying by its complex conjugate:

$$|C_{SC}(\omega)|^2_{m\ even} = \{\Delta T\}^2.\left\{\frac{\sin(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2})}\left[1-\frac{\cos(\omega\delta.\frac{\Delta T}{m})}{\cos(\omega.\frac{\Delta T}{2m})}\right]\right\}^2 \qquad (6.23)$$

Consequently, the power spectral density for the BOC signal (cosine sub carrier) including the code waveform is:

$$\Phi_{BOC3}(\omega) = \frac{\Delta T}{(1-2\delta)}.\left\{\frac{\sin(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2})}\left[1-\frac{\cos(\omega\delta.\frac{\Delta T}{m})}{\cos(\omega.\frac{\Delta T}{2m})}\right]\right\}^2 \qquad (6.24)$$

It is worthy to note that the spectrum can be modified by the presence of δ in the cosine function in the numerator. The power spectral density for the BOC modulation with a cosine sub carrier (with $\delta=0$) is:

$$\Phi_{BOC}(\omega) = \Delta T.\left\{\frac{\sin(\omega.\frac{\Delta T}{2}).\left\{\cos(\omega.\frac{\Delta T}{2m})-1\right\}}{\cos(\omega.\frac{\Delta T}{2m}).(\omega.\frac{\Delta T}{2})}\right\}^2 \qquad (6.25)$$

Care should be exercised in evaluating equations A2.23 to A2.25 at its singular values.

Case 4: The Fourier Transform of $c_{SC}(t)$ for m odd with a cosine sub carrier, as defined above in equation A2.5, is:

$$C_{SC}(\omega) = \int_{-\infty}^{\infty} c_{SC}(t).\exp(-j\omega t).dt$$

$$= \sum_{r=-(m-1)/2}^{+(m-1)/2}\left\{\int_{t=(r-\frac{1}{2})\Delta T/m}^{(r-\delta)\Delta T/m}(-1)^r.\exp(-j\omega t).dt + \int_{t=(r+\delta)\Delta T/m}^{(r+\frac{1}{2})\Delta T/m} -(-1)^r.\exp(-j\omega t).dt\right\}$$

$$= \frac{-2}{j\omega}\left[\left\{\cos(\omega\delta\frac{\Delta T}{m}) - \cos(\omega.\frac{\Delta T}{2m})\right\}\left\{\sum_r(-1)^r\exp(-j\omega r\frac{\Delta T}{m})\right\}\right] \qquad (6.26)$$

The final step in evaluating equation 6.26 concentrates on the summation which can be performed as before. With the summation denoted as U:

$$U = \sum_{r=-(m-1)/2}^{(m-1)/2}(-1)^r.\exp(-j\omega r\frac{\Delta T}{m})$$

$$= \left[\frac{x^{-m/2} - x^{+m/2}}{x^{-1/2} - x^{+1/2}}\right] \qquad (6.27)$$

$$= -(-1)^{-(m-1)/2}\left\{\cos(\omega\frac{\Delta T}{2})/\cos(\omega\frac{\Delta T}{2m})\right\}$$

Then:

$$C_{SC}(\omega) = \frac{-2.(-1)^{-m/2}}{j\omega}\left[\left\{\cos(\omega\delta.\frac{\Delta T}{m}) - \cos(\omega\frac{\Delta T}{2m})\right\}.\left\{\frac{\cos(\omega.\frac{\Delta T}{2})}{\cos(\omega.\frac{\Delta T}{2m})}\right\}\right] \qquad (6.28)$$

The power spectral density is formed from equation A2.22 by multiplying by its complex conjugate:

$$|C_{SC}|^2 = \{\Delta T\}^2.\left\{\frac{\cos(\omega.\frac{\Delta T}{2}).\left\{\cos(\omega.\frac{\Delta T}{2m}) - \cos(\omega\delta.\frac{\Delta T}{m})\right\}}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2 \qquad (6.29)$$

Consequently, the power spectral density for the BOC signal (cosine sub carrier) including the code waveform is:

$$\Phi_{BOC3} = \frac{\Delta T}{(1-2\delta)} \cdot \left\{ \frac{\cos(\omega.\frac{\Delta T}{2}).\left\{\cos(\omega.\frac{\Delta T}{2m}) - \cos(\omega\delta.\frac{\Delta T}{m})\right\}}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})} \right\}^2 \tag{6.30}$$

It is worth noting that the spectrum is modified by the presence of δ in the cosine function in the numerator. The power spectral density for the BOC modulation with a cosine sub carrier (with $\delta=0$) is:

$$\Phi_{BOC}(\omega) = \Delta T.\left\{ \frac{\cos(\omega.\frac{\Delta T}{2})}{\cos(\omega.\frac{\Delta T}{2m})} \cdot \frac{\left\{\cos(\omega.\frac{\Delta T}{2m}) - 1\right\}}{(\omega.\frac{\Delta T}{2})} \right\}^2 \tag{6.31}$$

Care should be exercised in evaluating equations 6.29 to 6.31 at any singular values.

Modified BOC Spectra

In the previous section, we have found formulae for the power spectral density for a family of sub carrier modulated spreading functions. The family ranges from the well known BOC sub carrier with $\delta=0$ to 3 level waveforms with $0<\delta<0.5$. Clearly, the sine functions in cases 1 and 2 containing the δ factor can be tuned to force a zero in the power spectral density at:

$$\omega.\frac{\Delta T}{2m}.(1-2\delta) = \pi \qquad f = \frac{m}{\Delta T(1-2\delta)} \tag{7.1}$$

The waveform modification may be made to either the sub-carrier waveform or to the spreading waveform itself. Both may have advantages in spectral control. In the following case, we consider initially possible changes to the waveform of the sub carrier. For transmission purposes, these usually do not impact the complexity of the Navigation Signal Generator Unit (NGSU) to a large degree, especially if the possibility of such requirements has been accounted for early in the design. However, such waveforms have an effect on the range of modulations available on a specific transmission frequency.

There are a variety of waveforms that can be used for the sub carrier ranging from the binary one of the BOC to the sine wave of the LOC. The spectrum control implied by the LOC sine wave, however, means that the satellite HPA's do not operate with high efficiency. Consequently, various digital representations have been considered including 3 and 5 level sine wave approximations. The 3-level approximation has been used in the 3-state BOC (ref 4) and the 5 level waveform is considered here as the resolved component of an 8-PSK satellite signal.

The Argand diagram for an 8-PSK signal is shown in figure 3.

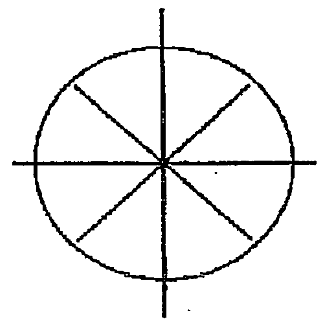


Figure 3 – Argand Diagram for 8-PSK Signal

From this figure, it can be seen that the waveforms of the I and Q components resolved onto their respective axes for the 8-PSK waveform are 5 level signals (with levels: $-1, -1/\sqrt{2}, 0, 1/\sqrt{2}, +1$). Such a stepped waveform approximates a sinusoid at sampling instants every 45 degrees. Similarly, a 3 state waveform results from a QPSK signal where there are dwells at the '0' level. In order to observe the effects of the dwell of zero, a 3 level sub carrier modulation of the BOC(2,2) type is plotted in figure 3 also for dwell periods of ($2\Delta T/8m$) at each transition of the sub-carrier (corresponding to a 45 degree dwell angle per half cycle of the sub carrier).

In the case of k-level BOC modulation, the rectangular code spreading waveform is further modulated by a k-level sub-carrier having an integer number of half cycles within the duration ΔT.

One way to model the sub carrier modulation is to revise the definition of the spreading waveform to $h_{oc}(t)$ through multiplication of the basic spreading waveform with the sub-carrier waveform:

$$h_{oc}(t) = h(t).c_{Ksc}(t)$$
$$\text{where } c_{Ksc}(t) \textit{ is the } \text{k-level sub-carrier waveform} \tag{7.2}$$

We set up the specific waveform for the non-zero interval of h(t) by means of the superposition of the 3 level waveform but with different transition times. For example a 5 level waveform for the sub carrier modulation is specified for 8 sub intervals within the period $\Delta T/m$.

$$c_{KSC}(t)=0 \qquad 0.\frac{\Delta T}{8m}<t<1.\frac{\Delta T}{8m}$$
$$c_{KSC}(t)=\frac{1}{\sqrt{2}} \qquad 1.\frac{\Delta T}{8m}<t<3.\frac{\Delta T}{8m}$$
$$c_{KSC}(t)=1 \qquad 3.\frac{\Delta T}{8m}<t<5.\frac{\Delta T}{8m} \qquad (7.3)$$
$$c_{KSC}(t)=\frac{1}{\sqrt{2}} \qquad 5.\frac{\Delta T}{8m}<t<7.\frac{\Delta T}{8m}$$
$$c_{KSC}(t)=0 \qquad 7.\frac{\Delta T}{8m}<t<8.\frac{\Delta T}{8m}$$

Equation 7.3, sets up repetitive binary waveforms, with 5 levels, and transitions at on a grid of intervals of ($\Delta T/8m$). This waveform is synthesised from two 3 level waveforms, the first with an amplitude of ($1/\sqrt{2}$) with transitions at ($\Delta T/8$) and ($7\Delta T/8$) and a second wave with amplitude of ($1-1/\sqrt{2}$) and transition times of ($3\Delta T/8$) and ($5\Delta T/8$).

Case 1: The Fourier Transform of $c_{SC}(t)$ for m even is, therefore:

$$C_{KSC}(\omega)=\int c_{KSC}(t).\exp(-j\omega t).dt \qquad (7.4)$$
$$=\rho_3 C_{3SC}(\omega,\delta_3)+\rho_5 C_{3SC}(\omega,\delta_5)$$

The ρ functions weight the two spectra (with phase components) for each section of the 5 level waveform with $\delta_1=0.125$ and $\delta_2=0.375$.

$$C_{KSC}=\frac{\Delta T.(-1)^{-m/2}}{-(\omega\frac{\Delta T}{2})}.\sin(\omega\frac{\Delta T}{8m}).\left[\left\{1+\sqrt{2}\cos(\omega\frac{\Delta T}{4m})\right\}.\left\{\frac{\sin(\omega\frac{\Delta T}{2})}{\cos(\omega\frac{\Delta T}{2m})}\right\}\right]$$

and the power spectral density then is

$$|C_{KSC}|^2=(\Delta T)^2.\left\{\frac{\sin(\omega.\frac{\Delta T}{8m}).(1+\sqrt{2}\cos(\omega\frac{\Delta T}{4m}).\sin(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2$$

(7.5)

This is similar to the expression for binary sub carrier modulation. The correction for the code auto-correlation function is $1/\Delta T$ and for the BOC waveform is ½.

$$\Phi_5=2\Delta T.\left\{\frac{\sin(\omega.\frac{\Delta T}{8m}).(1+\sqrt{2}\cos(\omega\frac{\Delta T}{4m})).\sin(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2$$

(7.6)

This is plotted in figure 3 also for the values of $\delta_1=0.125$ and $\delta_2=0.375$.

Case 2: m odd. The formulation proceeds similarly from the previous case, but with different limits, until the expressions start to simplify:

$$C_{KSC}(\omega)=\int_{-\infty}^{\infty} c_{KSC}(t).\exp(-j\omega t).dt \qquad (7.7)$$
$$=\rho_3 C_{3SC}(\omega,\delta_3)+\rho_5 C_{3SC}(\omega,\delta_5)$$

The ρ functions weight the two spectra (with phase components) for each section of the 5 level waveform with $\delta_1=0.125$ and $\delta_2=0.375$.

$$C_{KSC}(\omega)=\frac{-\Delta T.(-1)^{-m/2}}{(\omega\frac{\Delta T}{2})}.\sin(\omega\frac{\Delta T}{8m}).\left[\left\{1+\sqrt{2}\cos(\omega\frac{\Delta T}{4m})\right\}.\left\{\frac{\cos(\omega\frac{\Delta T}{2})}{\cos(\omega\frac{\Delta T}{2m})}\right\}\right]$$

and the power spectral density for a 5 levels

$$|C_{KSC}|^2=(\Delta T)^2.\left\{\frac{\sin(\omega.\frac{\Delta T}{8m}).(1+\sqrt{2}\cos(\omega\frac{\Delta T}{4m}).\cos(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2$$

(7.8)

This is also similar to the expression for binary sub carrier modulation. The correction for the code auto-correlation function is $1/\Delta T$ and for the BOC waveform is ½.

$$\Phi_5=2\Delta T.\left\{\frac{\sin(\omega.\frac{\Delta T}{8m}).(1+\sqrt{2}\cos(\omega\frac{\Delta T}{4m})).\cos(\omega.\frac{\Delta T}{2})}{(\omega.\frac{\Delta T}{2}).\cos(\omega.\frac{\Delta T}{2m})}\right\}^2$$

(7.9)

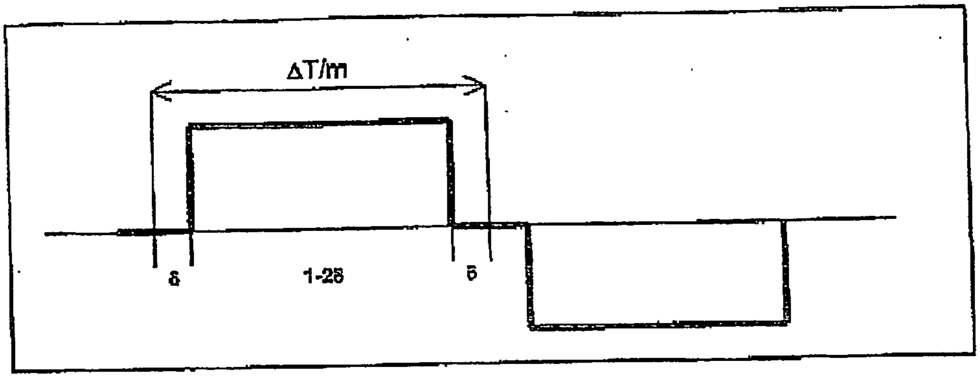


Figure A2.1 – Time response of 3 level sub carrier waveform with sine phasing

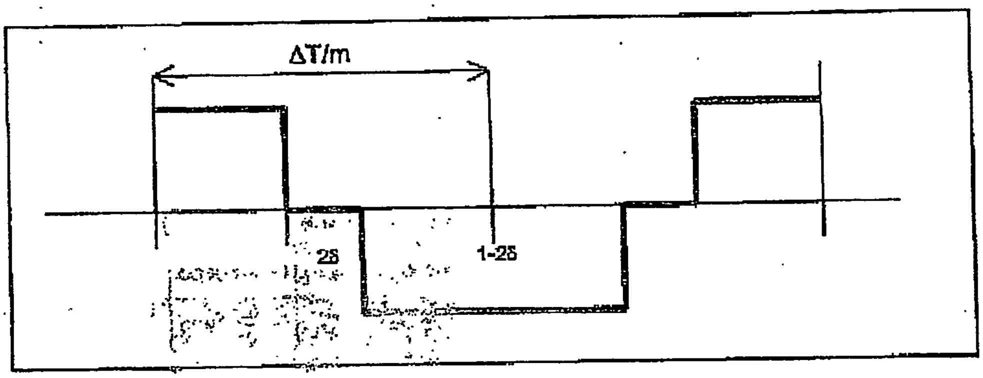
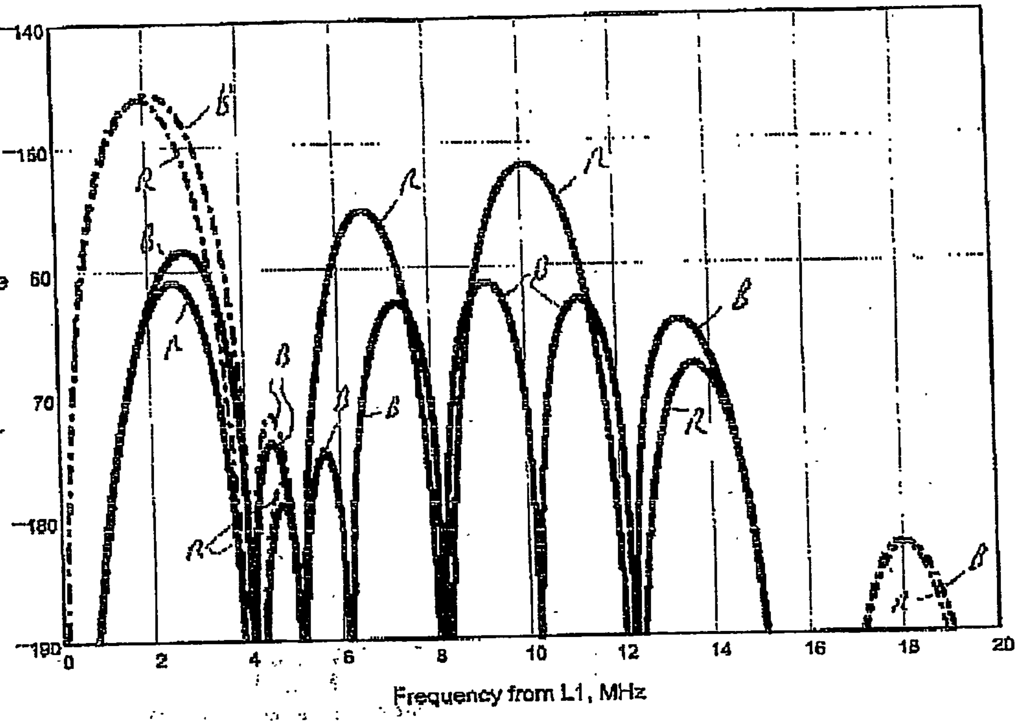


Figure A2.2 – Time response of 3 level sub carrier waveform with cosine phasing

Sub Carrier Frequency x 1.023MHz	Spreading Code Rate x 1.023MHz	Full SSC with itself (dBHz)	SSC with BOC(10,5) (dBHz)	Partial SSC BOC(10,5) ±5 MHz (dBHz)	Partial SSC BOC(10,5) ±(5-20) MHz (dBHz)	No of levels in BOC (2,2)	δ	No of levels in code
0	1	-61.86	[illegible]			-		2
0	2	-64.87	[illegible]			-		2
1	1	-64.87	-83.12			2		2
2	2	-67.88	-80.11	-81.48	-85.81	2		2
2	2	-67.66	-80.36	-80.59	-93.39	5	0.125 0.375	2
2	2	-67.82	-79.95	-80.77	-87.58	3	0.125	2
10	5	-73.16	-73.16	-91.37	-73.23	2		2
0	10	-71.86				-		2

Cross Power Spectrum – see text for legend.

R – Red
B – Blue



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